

## Answers to Practice Questions for Parts III and IV, Chapters 7-11

### Section 1. Multiple Choice Questions

1. C); 2. B); 3. A); 4.A); 5. B); 6. A); 7. B); 8. A); 9. B); 10. A); 11. B); 12. A); 13. B); 14. A); 15. B); 16. B); 17.A); 18. C); 19. D); 20. A); 21. B); 22. C); 23. C); 24. B); 25 D).

### Section 2. Problems

#### Chapter 7

##### 7.1

a) For an economy with technological progress the general growth accounting equation for growth in aggregate output is:

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1 - \alpha) \frac{\Delta L}{L}$$

In this case, we are told that:

$$\frac{\Delta K}{K} = \frac{\Delta L}{L}$$

Thus, by substitution:

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta L}{L} + (1 - \alpha) \frac{\Delta L}{L} = \frac{\Delta A}{A} + \frac{\Delta L}{L}$$

It is also always the case that the growth rate of output **per worker** is equal to the difference between the growth rate of aggregate output and the growth rate of the labour force:

$$\frac{\Delta(Y/L)}{(Y/L)} = \frac{\Delta Y}{Y} - \frac{\Delta L}{L}$$

Now, we are told that:

$$\frac{\Delta Y}{Y} = 0.03 \text{ and } \frac{\Delta(Y/L)}{(Y/L)} = \frac{\Delta Y}{Y} - \frac{\Delta L}{L} = 0.01.$$

Thus,  $\frac{\Delta L}{L} = \frac{\Delta K}{K} = 0.02$ ;  $\frac{\Delta A}{A} = 0.01$ .

In this economy, the labour force and capital stock are both growing at 2% per year while total factor productivity ( $A$ ) is increasing at 1% per year.

b)

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1 - \alpha) \frac{\Delta L}{L}$$

$$= 0.01 + (1/3)(0.04) + (2/3)(0.01) = 0.03$$

Thus, there is no change in the rate of growth of aggregate output (Y) which remains at 3%. However, the rate of growth of per capita output (or output per worker) increases to 2% per year:

$$\frac{\Delta(Y/L)}{(Y/L)} = \frac{\Delta Y}{Y} - \frac{\Delta L}{L} = 0.03 - 0.01 = 0.02$$

## 7.2

a)

Year	Capital per worker ( $k$ )	Output per worker ( $y$ )
1	\$90,000	\$30,000
2	<b>\$91,500</b>	<b>\$30,249</b>

Recall the general equation for change in the amount of capital per worker (for an economy without population growth or technological change):

$$\Delta k = sf(k) - \delta k$$

In this case, between Year 1 and Year 2:

$$\Delta k = 0.2(\$30,000) - 0.05(\$90,000) = \$6,000 - \$4,500 = \$1,500$$

Thus, in Year 2:  $k = \$90,000 + \$1,500 = \$91,500$  and  $y = 100\sqrt{\$91,500} = \$30,249$ .

b) In steady-state equilibrium capital per worker is constant from one year to the next:

$$\Delta k = sf(k) - \delta k = 0$$

$$\text{Or, } sf(k) = \delta k$$

In this case,

$$0.2(100\sqrt{k}) = 0.05k$$

$$\sqrt{k} = 20 / 0.05 = 400$$

$$k^* = \$160,000$$

$$y^* = 100\sqrt{k^*} = \$40,000.$$

c) The golden-rule level of capital per worker, or  $k^*_{gold}$ , is the value of  $k^*$  for which steady-state consumption per worker ( $c^*$ ) is maximized. Steady-state consumption per worker is equal to the difference between (steady-state) output per worker and (steady-state) investment per worker. In this case, steady-state investment per worker is equal to depreciation per worker. Thus:

$$c^* = y^* - i^* = f(k^*) - \delta k^*$$

$c^*$  attains its maximum value at the level of  $k^*$  at which the marginal product of capital (MPK), which is the first derivative (or slope) of the per capita production function,  $[f(k^*)]$ , is equal to the depreciation rate ( $\delta$ ), which is the first derivative (or slope) of the depreciation function ( $\delta k^*$ ):

$$\text{MPK} = \delta$$

$$\frac{50}{\sqrt{k^*}} = 0.05$$

$$k_{gold}^* = \$1,000,000; y_{gold}^* = 100\sqrt{k_{gold}^*} = \$100,000$$

d) In steady-state equilibrium with labour force growth at rate  $n$ :

$$\Delta k = sf(k) - (\delta + n)k = 0$$

$$\text{or } sf(k) = (\delta + n)k$$

In this case,

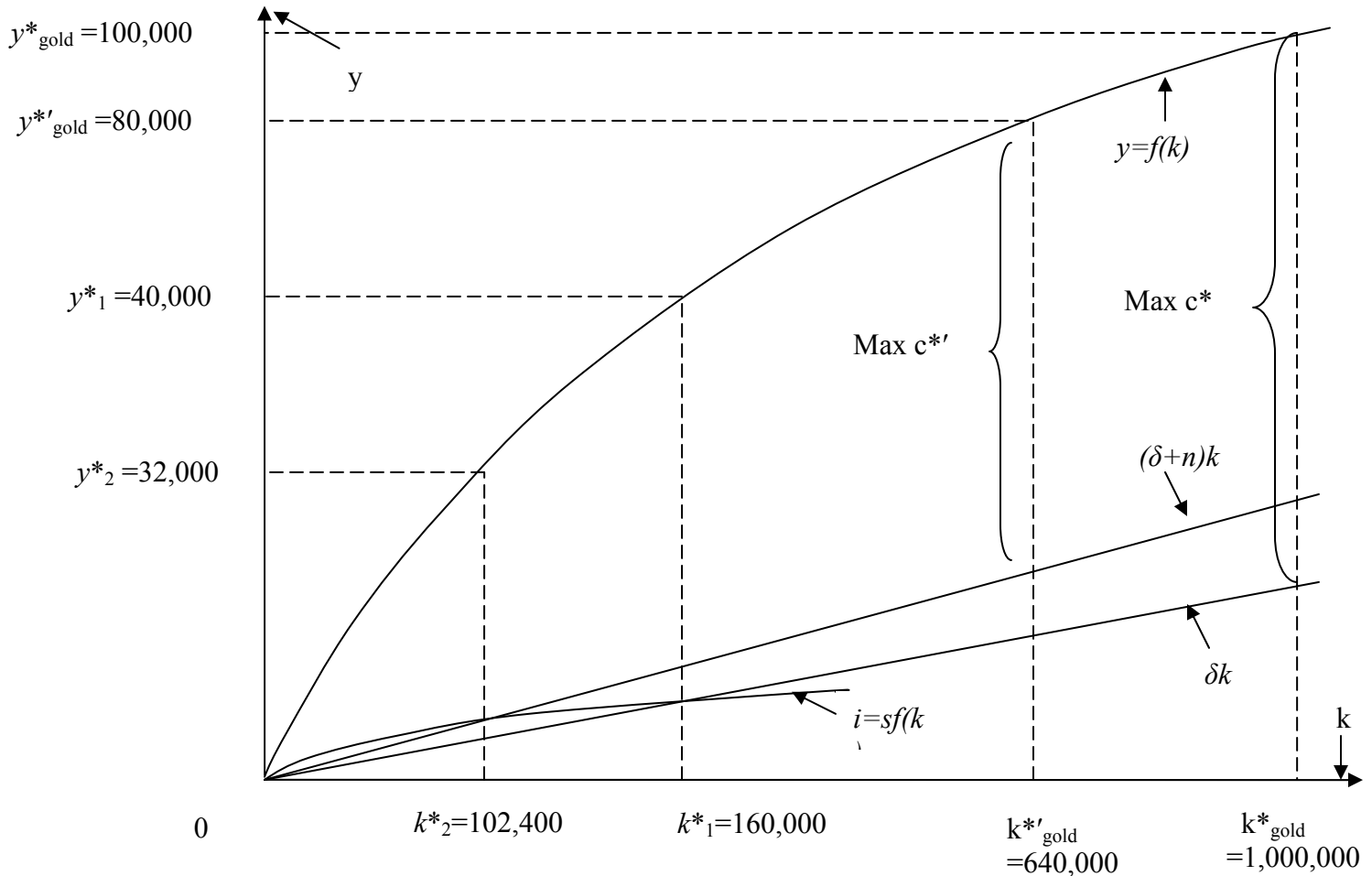
$$0.2(100\sqrt{k}) = (0.05 + 0.0125)k$$

$$\sqrt{k} = 20 / 0.0625 = 320$$

$$k^* = \$102,400$$

$$y^* = 100\sqrt{k^*} = \$32,000.$$

Starting from a steady-state with zero population growth, an increase in the labour force growth rate from zero to 1.25% increases the break-even level of investment per worker (the level required to keep capital per worker constant). Break-even investment per worker now exceeds actual investment per worker at the original steady-state level of capital per worker. Capital per worker then falls decreasing the gap between the amount of break-even investment per worker and the amount of actual investment per worker until a new steady-state equilibrium is reached with a lower level of capital per worker. A reduction in the steady-state level of capital per worker brings a reduction in the steady-state level of output per worker.



NOTE: Diagram **not** drawn to scale.

e) The golden-rule level of capital per worker, or  $k^*_{gold}$ , is the value of  $k^*$  for which steady-state consumption per worker ( $c^*$ ) is maximized. Steady-state consumption per worker is equal to the difference between (steady-state) output per worker and (steady-state) investment per worker. In this case, steady-state investment per worker is equal to **break-even investment per worker**. Thus:

$$c^* = y^* - i^* = f(k^*) - (\delta + n)k^*$$

$c^*$  attains its maximum value at the level of  $k^*$  at which the marginal product of capital (MPK), which is the first derivative (or slope) of the per capita production function,  $[f'(k^*)]$ , is equal to the **depreciation rate plus the growth rate of the labour force**  $(\delta + n)$ , which is the first derivative (or slope) of the **break-even investment function**  $[(\delta + n)k^*]$ :

$$MPK = \delta + n$$

$$\frac{50}{\sqrt{k^*}} = 0.0625$$

$$k^*_{gold} = \$640,000; y^*_{gold} = 100\sqrt{k^*_{gold}} = \$80,000$$

The golden-rule steady-state value of capital per worker is lower in this case than in the previous case [ part c) above] because consumption per worker is maximized at a higher value of the marginal product of capital ( $MPK = 0.0625 > 0.05$ ). But, given diminishing marginal productivity of capital, a higher value for MPK occurs at a lower level of capital per worker. Hence,  $k^*_{\text{gold}}$  must be lower.

## **Chapter 8**

**8.1** In the Solow growth model with (labour-augmenting) technological change a steady-state is attained when:

$$sf(k) = (\delta + n + g)k$$

and,  $\frac{\Delta Y}{Y} = n + g$

In this case we are told that:

$$\frac{\Delta Y}{Y} = 0.03; \delta = 0.04; \frac{k}{y} = 2.5$$

Thus,

$$\begin{aligned} sf(k) &= (\delta + n + g)k \\ &= (0.04 + 0.03)2.5y \\ sy &= 0.175y \\ s &= 0.175 \text{ (17.5\%)} \end{aligned}$$

The implied saving rate for the US is 17.5% of income.

## **Chapter 10**

### **10.1**

$$E = C + I + G$$

$$C = 140 + 0.60(Y - T)$$

$$I = \bar{I} = 170$$

$$G = \bar{G} = 210$$

$$T = \bar{T} = 200$$

$$Y = E = C + I + G = 140 + 0.60(Y - 200) + 170 + 210$$

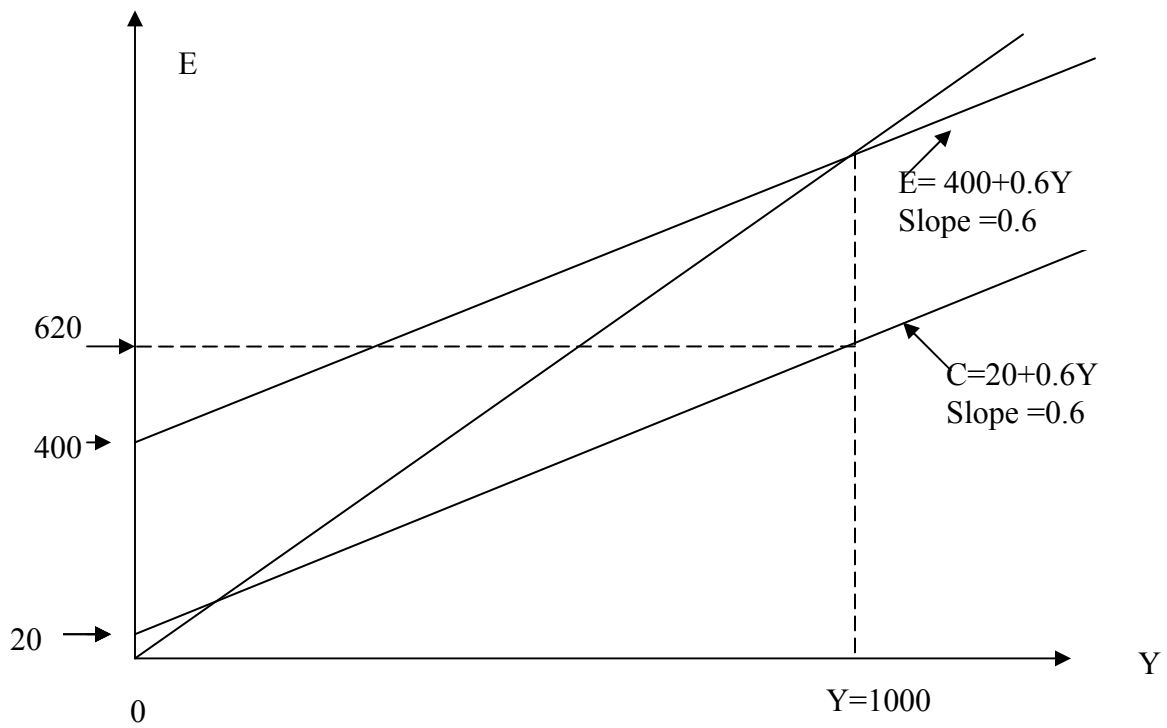
$$Y = 400 + 0.6Y$$

$$0.4Y = 400$$

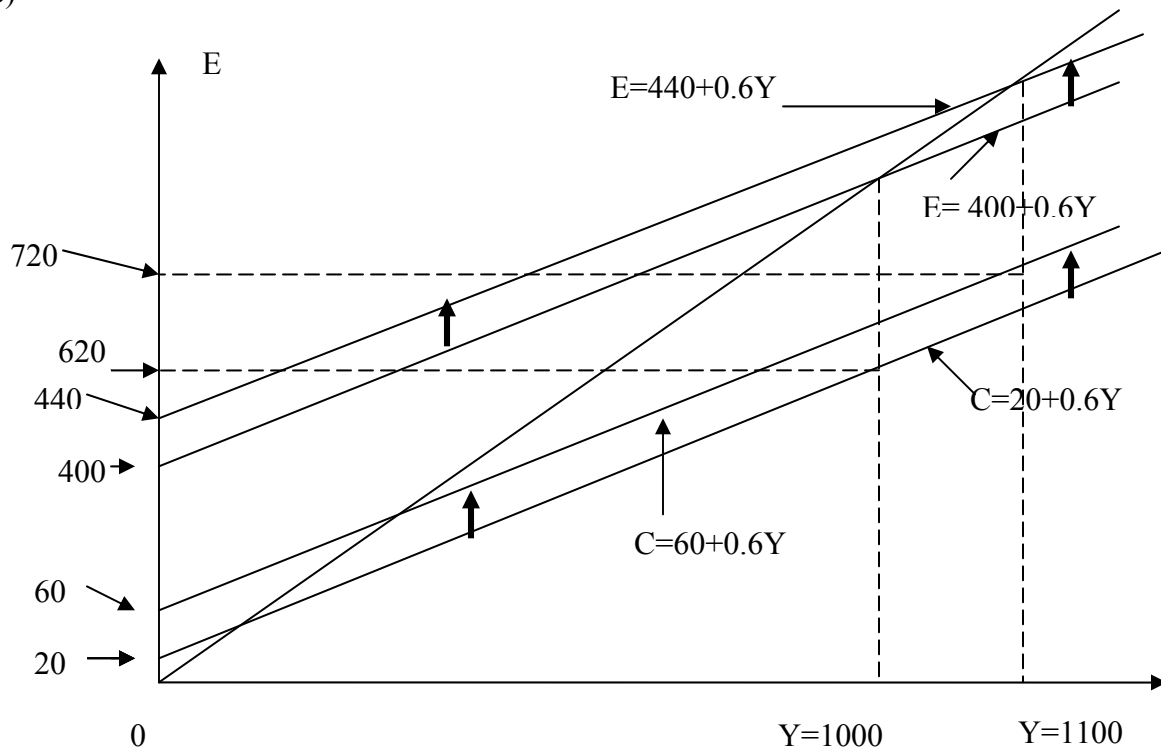
$$Y = 1000$$

$$C = 140 + 0.6(1000 - 200) = 620$$

b)



c)



Thus, the increase in consumer confidence increases consumption spending at each level of  $Y$  by \$40 billion shifting the consumption function vertically upwards by \$40 billion. The aggregate expenditure line shifts vertically upwards by \$40 billion. Equilibrium income rises by \$100 billion which is equal to the change in autonomous spending (+40) multiplied by the autonomous spending multiplier [ $1/(1-MPC) = 1/(1-0.6) = 2.5$ ]. [The sequence of events is explained at the bottom of Ch 10 Pg. 6 in the Lecture Notes (substitute an increase in (autonomous) consumption for the increase in  $G$ .)]

## 10.2

The IS curve is a line joining combinations of  $r$  and  $Y$  for which there is equilibrium in the output market of the economy ( $Y = E$ ) for given values of the fiscal policy variables ( $G$  and  $T$ ). Thus, along the IS curve:

$$\begin{aligned} Y &= E \\ &= C(Y-T) + I(r) + G \end{aligned}$$

Now we are told that the consumption functions are identical for economies A and B and the values of  $T$  and  $G$  are the same in both economies. Thus, any differences between the IS curves in the two economies must be the result of differences in their investment functions.

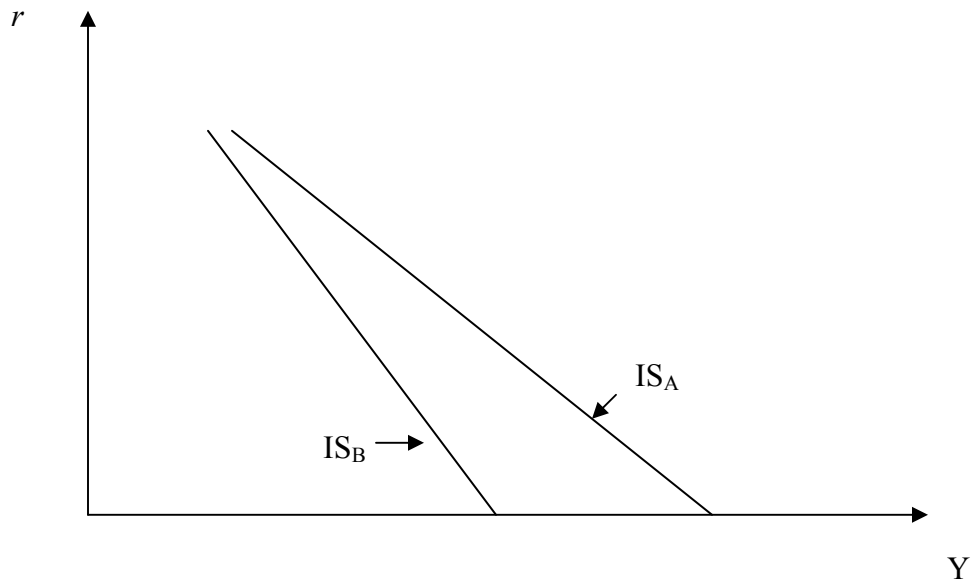
By inspection of the equations for  $I_A$  and  $I_B$  we can see that:

$$I_A = 100 - 500r > I_B = 60 - 400r > 0, \text{ for all } r < 0.15 \text{ (15\%)}$$

Thus, for any real interest rate within the range we would normally expect to observe (such that investment is positive in both economies), investment spending will be **higher in economy A** than in economy B and hence, at a given interest rate, total expenditure ( $E$ ) will be higher in A than in B resulting in a higher level of output ( $Y$ ) to maintain output market equilibrium ( $Y=E$ ). It follows that the IS curve for economy A will lie everywhere **to the right of** the IS curve for economy B (for  $r < 0.15$ )

Second, in economy A investment spending is **more responsive** to a change in real interest rate than is investment in economy B. Specifically, a one percentage point rise in  $r$  (e.g from 0.04 to 0.05) will reduce  $I_A$  by \$5 billion while reducing  $I_B$  by only \$4 billion. In economy A each one percentage point increase in  $r$  will result in a larger decrease in  $I$  and require a larger decrease in  $Y$  to keep  $Y = E$  than will result from a one percentage point increase in  $r$  in Economy B. Thus, the IS curve of economy A is **flatter** than the IS curve of economy B

In summary, the two IS curves would resemble those shown below:

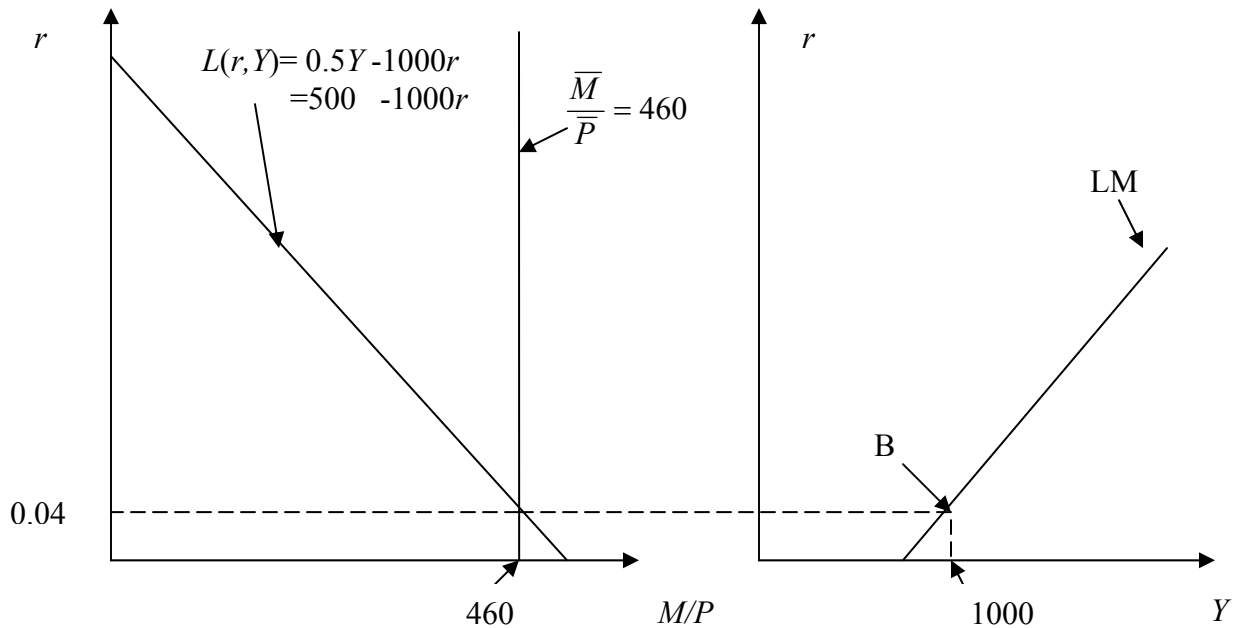


### 10.3.

An LM curve joins all combinations of  $r$  and  $Y$  for which the money market is in equilibrium for a given (real) money supply. Along any specific LM curve:

$$(M/P)^d = L(r, Y) = \bar{M}/\bar{P} \quad \text{for given (constant) } \bar{M}/\bar{P}$$

- i) Combinations B, C and E are on an LM curve. In each of B, C and E the combination of  $Y$  and  $r$  generates an aggregate demand for money which exactly matches the real supply of money associated with that combination.
- ii) Combinations A and D are not on an LM curve because in each of these cases there is either an excess demand or excess supply of money ( for combination A there is an excess **demand** of 10 and for combination D there is an excess **supply** of 20).
- iii) Combinations B and C are on the **same** LM curve because in both cases the demand for money equals the supply of money and the supply of money is the **same (460)**.



## Chapter 11

### 11.1

We know that at point A (initial equilibrium):

$$Y = E = C(Y - \bar{T}) + I(r) + \bar{G} = 100$$

At point B (new equilibrium):

$$Y = E = C(Y - \bar{T}) + I(r) + \bar{G} = 120$$

Between points A and B:

$$\Delta Y = \Delta E = \Delta C(Y - \bar{T}) + \Delta I(r) + \Delta \bar{G} = 20$$

Now the change in consumption spending between points A and B is equal to the change in income multiplied by the MPC:

$$\Delta C = MPC \Delta Y = MPC(20)$$

The MPC can be calculated from the information given as follows. Recall that a given increase in government spending shifts the IS curve to the right by a **horizontal distance** equal to the change in  $G$  multiplied by  $(1/(1-MPC))$ . In this case a \$10 billion increase in  $G$  causes a horizontal shift in the IS curve of \$40 billion ( $140 - 100$  at interest rate  $0.02$ ). Hence:

$$\frac{\Delta \bar{G}}{(1 - MPC)} = 40$$

$$(1 - MPC) = \frac{\Delta \bar{G}}{40} = \frac{10}{40} = 0.25$$

$$MPC = 0.75$$

Thus,  $\Delta C = MPC\Delta Y = 0.75(20) = 15$  (\$ billion)

The change in investment spending between points A and B can then be found residually as follows:

$$\Delta Y = \Delta E = \Delta C(Y - \bar{T}) + \Delta I(r) + \Delta \bar{G} = 20$$

$$\text{Thus } \Delta I(r) = 20 - \Delta C(Y - \bar{T}) - \Delta \bar{G}$$

$$= 20 - 15 - 10$$

$$= -5$$

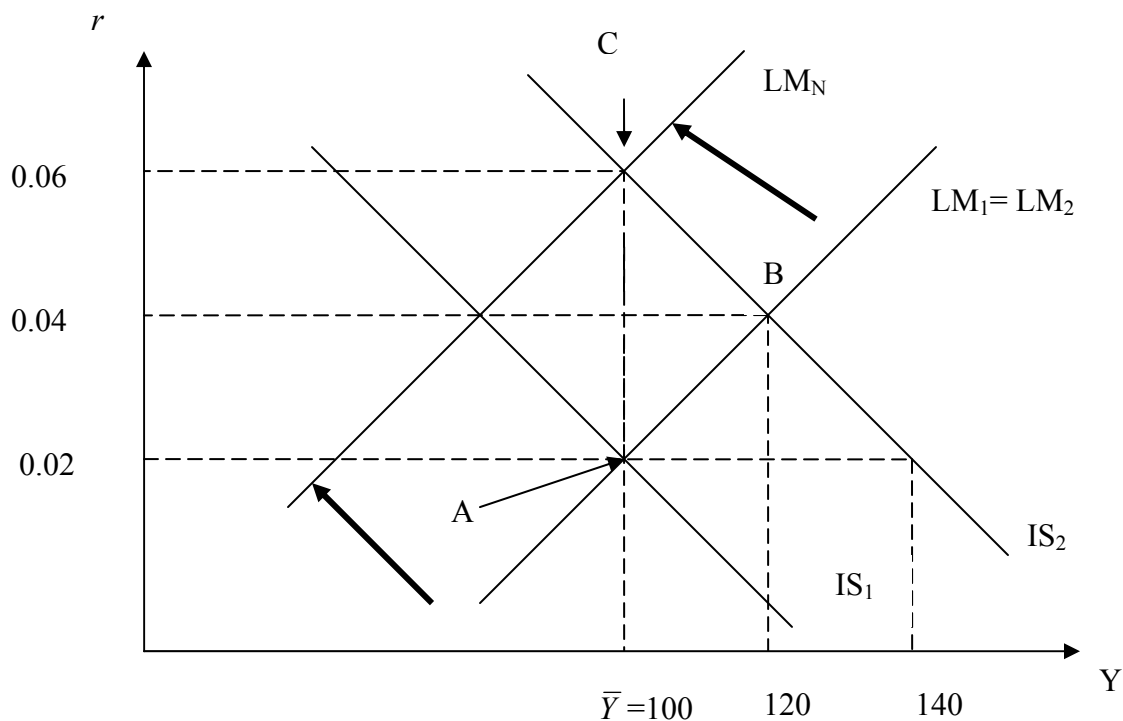
[ The process of adjustment of Y and r from point A to point B is explained in the Lecture Notes, Ch.11 Pg. 2.]

b) The short-run effect of this increase in government spending is to increase Y from 100 to 120 which is above the full-employment, or natural, level of output ( $\bar{Y} = 100$ .) As a consequence, in the long-run prices will rise and the real supply of money will fall shifting the LM curve back to the left (with no shift in the IS curve). The interest rate will rise above its short-run level of 4% reducing investment spending, overall spending and output. This process will continue until output has fallen to its long run equilibrium level of 100 ( $= \bar{Y}$ ) at which point prices are stable.

The long-run equilibrium rate of interest will be 0.06(6%). We know this because along  $IS_2$  the level of Y falls by 20 for each two percentage point rise in interest rate: an increase in the interest rate from 2% to 4% reduces Y from 140 to 120. Hence, a rise in the interest rate from 4% to 6% is required to reduce Y from its short-run equilibrium level of 120 to its long-run equilibrium level of 100. The diagram below shows the adjustment from the short-run equilibrium at point B to the new long-run equilibrium at point C.

Between points A and C the composition of output (Y) has changed as follows: govt. spending (G) has increased by \$10 billion; investment spending (I) has fallen by \$10 billion because of the increase in the real interest rate from 2% to 6%; and consumption spending (C) has not changed because aggregate disposable income (Y-T) has not changed with no changes in either Y or T.

[ The process of adjustment of Y and r from point B to point C is explained in the Lecture Notes, Ch.11 Pg. 10.]



c) At the initial short-run equilibrium (point A):

$$\begin{aligned} (M/P)^d &= Y - 1000r = \bar{M} / P_1 \\ 100 - 1000(0.02) &= 80/P_1 \\ 100 - 20 &= 80/P_1 \\ P_1 &= 1.0 \end{aligned}$$

After the increase in  $G$  the economy reaches a new short-run equilibrium at point B where the money market is in equilibrium with an unchanged money supply both nominal and real:

$$\begin{aligned} \text{At point B: } (M/P)^d &= Y - 1000r = \bar{M} / P_1 \\ 120 - 1000(0.04) &= 80/1 \\ 120 - 40 &= 80 \end{aligned}$$

In the long run the price level rises reducing the real supply of money (with no change in the nominal money supply) until the money market is once again in equilibrium at point C where:

$$\begin{aligned} (M/P)^d &= Y - 1000r = \bar{M} / P_2 \\ 100 - 1000(0.06) &= 80/P_2 \\ 40 &= 80/P_2 \\ P_2 &= 2.0 \end{aligned}$$